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Abstract

We analyze mutual insurance arrangements (policies based on risk-sharing among a pool of policyholders) when consumers choose a self-insurance effort, that is an effort decreasing the size of any loss occurring. We consider both cooperative and non-cooperative strategies in the effort choice. Cooperation among policyholders leads to the full internalization of the positive impact the effort exerts on the premium. We show that, for an infinite size of pool, with cooperation first-best efficiency is achieved. Moreover, cooperation is sustained as an equilibrium in a repeated interaction game for a sufficiently low size of pool. An interesting implication of our results is that a cooperative mutual policy can dominate a stock insurance contract. Simulations show that mutual insurance with cooperation as an equilibrium dominates a second-best stock-type insurance policy even when pool size is low.

JEL classification: D82, I11, I18.

Keywords: Mutual arrangement, self-insurance, positive externality on the insurance premium, cooperation.

1 Introduction

The recent economic crisis discredited the management model of many of the world's largest insurance companies[1] and caused widespread disenchantment among consumers. This climate gave alternative insurance schemes such as *mutual* arrangements an incredible opportunity to market their own

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[1] A good example is the American International Group (AIG) which is currently a leading international insurance organization serving customers in more than 130 countries. It recently planned to sell a significant number of its businesses, looking to streamline operations due to financial difficulties. For an overview of insurance companies in distress, see the *Global Financial Stability Report* released by the International Monetary Fund.

business model and benefit from customers' desire to conduct their economic choices in a more ethical and cooperative way. Having the consumer as the central stakeholder may offer mutual insurance companies a significant advantage over stock insurance companies, as a customer-driven business[2].

In a mutual arrangement, consumers purchase *a participating policy*, at the same time becoming the owners of the insurance firm (see Picard 2009). Members of the mutual risk agreement contribute whatever amount is needed yearly to meet the losses insured by the pool. This is usually in the form of an initial (partial) contribution followed by later 'calls', if required, to maintain the common fund. This means in practice that the premium of a mutual policy is *random* and, as a consequence, that policyholders in mutual schemes always face aggregate risk. Note that, in contrast, the premium of a stock insurance policy is always fixed. Since Borch (1962) theorists have evaluated the difference risk-sharing performance of stock and mutual insurance companies (see Eeckhoudt and Kimball, 1992; Doherty and Dionne,1993). Stock insurance companies can spread macroscopic risk over all investors in the capital market and are substantially risk-neutral, while mutual companies are consistently defined by risk-pooling which can spread macroscopic risk only across its membership. This necessarily implies that other advantages offered by risk-pooling arrangements are investigated in the insurance literature to offset mutual firms' inferior risk-sharing capacity. Mayers and Smith (1986, 1988) showed that mutual forms of organization are sometimes efficient at controlling expropriator behavior by owners and managers. Smith and Stutzer (1990) analyzed how participating policies can be used as a self-selection device when there is exogenous aggregate uncertainty. They formalize how, in the presence of macroscopic risk, the participating nature of risk-pooling renders the mutual insurance policy an efficient risk-sharing process. Smith and Stutzer (1995) showed that the further inclusion of the impact of moral hazard does not change the findings of their previous model. Instead, Ligon and Thistle (2005) offer an alternative reasons for the existence of mutual insurance and explain the context that enables mutuals and stock insurers to coexist. They demonstrate that, under certain conditions, a separating equilibrium exists in which high-risk type agents form large mutuals while low risk agents form small mutuals[3]. The small mutual firms (due to lower risk-sharing among members) thus solve the problem of adverse selection.

We focus on an alternative potential advantage characterizing mutual agreements: the opportunity to internalize a positive externality on the insurance premium exerted by a self-insurance measure available to consumers. In particular, we analyze a mutual agreement among n identical consumers faced with choosing a *self-insurance* measure. The consumers' preventive effort, which is not contractible, reduces the amount of any loss that may occur. We partially borrow from Lee and Ligon (2001) who, contrary to us, analyze a non-cooperative solution in the case of *self-protection*, i.e. the consumers' effort decreases their loss probability. Despite the presence of ex-ante moral hazard, they show that, in the mutual agreement, full nominal coverage is optimal and implies a positive loss-prevention effort.

We first prove that, in a mutual agreement, full coverage is also optimal when a self-insurance measure is available to consumers. Unlike Lee and Ligon (2001), we also focus on the cooperative option on choice of effort and compare cooperative and non-cooperative outcomes. In particular, we investigate how policyholders *internalize the positive impact of the effort on the policy premium* in both the cooperative and the non-cooperative (equilibrium). By choosing the non-cooperative strategy, each policyholder takes into account only the effect of his/her own effort on the premium when

[2]In its most recent annual survey of mutual insurers' business, the International Cooperative and Mutual Insurance Federation (ICMIF hereafter) found that the market share of mutual insurers grew from 23.3% in 2007 to 26% in 2008. Total premiums for the mutual and cooperative industry increased by 3.3% compared with 2007.

[3]The conditions under which this separating equilibrium exists are analogous to those proposed by the standard model à la Rothschild and Stiglitz (1976).

he/she is one of the members of the pool experiencing the loss, resulting in the non-cooperative symmetric Nash equilibrium. In contrast, cooperation implies a complete internalization of the impact of the effort (exerted by all policyholders) on the random premium; in other words, policyholders in the pool act as a single individual (whose payoff is the aggregation of the members' expected utility functions), so a single effort is chosen. Obviously the cooperative outcome dominates the non-cooperative equilibrium in terms of consumers' surplus.

We show that, as pool size rises to infinity (and the maximum level of risk-sharing is reached), the social welfare achieved by the cooperative outcome in the mutual agreement exactly replicates the first-best allocation. Of course mutual agreements do not always deliver cooperation. However, we identify conditions under which cooperation can be sustained as an equilibrium within a repeated interaction game with punishment strategies. Not surprisingly, we show that, if the pool size is sufficiently low, cooperation is enforceable.

In the last part of the paper we compare a stock-like insurance policy with the cooperative and the non-cooperative mutual policy obtained by solving our model. Note that a consumer purchasing a stock insurance policy never internalizes the positive impact of effort on the premium, since the premium is fixed and taken as given.

An interesting consequence of the fact that mutual arrangements under cooperation replicate the first-best allocation when pool size is infinite is that, if the number of policyholders in the pool is sufficiently large, a cooperative mutual policy dominates a second-best stock-type insurance contract, a new finding with respect to what the existing literature suggests (see Ligon and Thiestle 2008 and Arnott and Stiglitz 1988). Importantly, our numerical simulations show that the size of pool required for a cooperative mutual policy to dominate a second-best stock-type insurance contract is fairly low and this threshold guarantees the tenability of cooperation as an equilibrium. This result is relevant, indicating not only that cooperation can be sustained as an equilibrium, but also that cooperation is an equilibrium when the size of the mutual is such that the mutual policy dominates the second-best stock-type insurance policy.

The structure of the article is as follows. Section 2 discusses the first-best with self-insurance and the second-best stock-type policy. Section 3 presents a basic analysis of the mutual agreement. Sections 4 and 5 describe the non-cooperative equilibrium and the cooperative outcome in the mutual agreement, respectively. The relationship between pool size and efficiency and how to implement cooperation are discussed in Section 6. Mutual and stock-like insurance policies are compared and numerical simulations proposed in Section 7. Concluding remarks follow in the last section.

2 The model

The economy is composed of n identical individuals. They have initial wealth w and face the probability p of incurring a loss of size $L(e)$ with independently and identically distributed risks. The loss $L(e)$ is a function of individuals' nonnegative effort level e such that $L'(e) < 0$. From Ehrlich and Becker (1972), a consumer's effort decreasing the amount of the loss is a self-insurance measure. The level of effort e is exerted before the risk is realized. Each individual's utility is represented by a strictly increasing von Neumann-Morgenstern utility function $U(w)$ which is differentiable at least twice, with $U'(w) > 0$, $U''(w) < 0$. It is assumed to be additively separable in utility from money and in cost of effort such that $C(e)$ denotes the disutility of effort with $C'(e) > 0$ and $C''(e) > 0$.

2.1 First-best

Here we show the first-best of the previously depicted situation. With consumers' effort observable and contractible, the insurance firms solve the following program:

$$\max_{e,q} EU = pU\left(w - L(e) - pqL(e) + qL(e)\right) + (1-p)\,U\left(w - pqL(e)\right) - C(e) \tag{1}$$

where q is the cost-sharing parameter, with $0 \leq q \leq 1$. Consumers receive $qL(e)$ in the event of loss and the fair premium is $pqL(e)$. Let us define net consumption in the two states of nature as W_L when loss occurs and W_0 with no loss, respectively. The optimal value of the cost-sharing parameter q is as follows:

$$q^{FB} : p(1-p)U'\left[W_L\right]L(e) = p\,(1-p)\,U'\left[W_0\right]L(e) \tag{2}$$

Therefore, $U'\left[W_L\right] = U'\left[W_0\right]$ implying full coverage $\left(q^{FB} = 1\right)$, so that net consumption in the two possible states of the world is the same: $W_L = W_0 = W = w - pL(e)$. Consequently, the optimal choice of effort is derived as follows:

$$e^{FB} : U'(w - pL(e))\left(-pL'(e)\right) = C'(e) \tag{3}$$

e^{FB} corresponds to the first-best level of effort, useful in the next sections for further comparisons.

The left hand side of (3) shows the marginal benefit and the right hand side the marginal cost of the effort. Note that in first-best, consumers perfectly internalize the beneficial effect of the effort on the premium. In particular, they take into account that a higher effort, by decreasing the premium, has a positive impact on marginal utility in both the possible states of nature. Marginal benefit is increasing in p and in the efficiency of the self-insurance technology $-L'(e)$. Of course in first-best, consumers' welfare is maximized and corresponds to:

$$EU^{FB} = U\left(w - pqL(e^{FB})\right) - C(e^{FB}) \tag{4}$$

2.2 Informational structure

The function $L(e)$ is deterministic and is ex-post observable by the insurers so that, in principle, the insurers could perfectly infer the effort e from the realization of any loss occurring. However in the following, we assume that mutual and stock insurers are constrained to use linear contracts. In particular, we focus on the case where the contract requires a constant cost-sharing parameter $q \leq 1$ (like the one we used in first-best above). Note that this type of contract is typical in the health insurance market, where it leads to the standard trade-off between optimal risk sharing and incentives, as with ex-post moral hazard.[4] Although in our model consumers choose the effort ex-ante (since the effort is a self-insurance measure), whereas in ex-post moral hazard models consumers choose ex-post (once the loss is realized), the trade-off between optimal risk sharing and incentives obviously arises in our setting too (see the contract described below).

As for the insurance contract, a *constant and linear* cost-sharing parameter implies that the effort e is not contractible. Thus, in the model, consumers choose the effort *given* the insurance contract (q is not a function of e) and insurers anticipate how the contract they offer influences consumers' choice of effort.

[4]For an economic analysis of ex-post moral hazard see, for example, Zeckhauser (1970); an empirical analysis can be found in Manning et al. (1987).

2.3 The second-best (stock-like) insurance contract

In this subsection we consider a competitive market with stock-insurers offering contracts to consumers when the effort is not contractible. The timing of actions is as follows: first, the insurance firms propose the contract; second, the consumers accept the contract and choose the effort level; finally the risk is realized.

In the second-best contract, denoted (P, q), P is the premium and q, as before, is the cost-sharing parameter. Again consumers receive $qL(e)$ in the event of loss. Since we assume a competitive stock insurance market the premium is fair: $P = pqL(e)$.

Given the insurance contract (P, q), the representative consumer's expected utility is:

$$EU^{SB} = pU\left[w - L(e) - P + qL(e)\right] + (1-p)\,U\,(w - P) - C(e) \tag{5}$$

Note that the optimal effort level is:

$$\arg\max_{e} EU^{SB}(e; P, q)$$

so that the effort level is calculated *given* the contract (P, q). Consumer's optimal choice thus verifies:

$$e^{*SB}(q) : -(1-q)L'(e)pU'(W_L) = C'(e) \tag{6}$$

Obviously, if $q = 1$ then the effort is zero so that full insurance is not the optimal policy $(W_L \neq W_0)$. By comparing (3) and (6) we observe that in the latter FOC consumers do not internalize the positive impact the effort has on the premium. In fact, in the l.h.s. of (6) only the positive impact the effort has on any loss occurring (and net consumption is W_L), is taken into account. As we will show in the next section, the FOC is different in the case of mutual insurance since the consumers (at least partially) internalize the positive impact of the effort on the premium.

In the first step, insurance firms solve the following program:[5]

$$\begin{aligned} &\max\nolimits_q EU^{SB} = pU\left(W_L\right) + (1-p)\,U\left(W_0\right) - C(e) \\ &s.t. : P = pqL(e) \\ &-(1-q))L'(e)pU'(W_L) - C'(e) = 0 \qquad (IC) \end{aligned} \tag{7}$$

where $W_L^{SB} = w - L(e) - P + qL(e)$, $W_0^{SB} = w - P$ and, (IC) is the consumer's incentive constraint. It is well known that when program (7) is solved, the optimal level of coverage q is found to be lower than 1 (partial coverage), which implies that the usual trade-off between optimal incentives and risk-sharing arises.

3 Mutual insurance

Suppose that the n identical individuals enter into a mutual arrangement such that the indemnity paid by the mutual insurance for an individual's loss is $qL(e)$. Again q is the percentage of the loss

[5]The second-best contract presented in this section is not, obviously, the best contract that stock insurers can offer to consumers given our assumptions. However, as we explained in Subsection 2.2, for tractability reasons and since it is the most typical contract offered by stock insurers in the real world, we focus on *linear* contracts. In particular, we will use this second-best contract to compare stock insurance and mutual insurance policies.

reimbursed to the policyholder. Let us call K the number of consumers that experience the loss out of the n identical consumers in the pool: $K \in \{0, ..., n\}$. The peculiarity of mutual insurance is that the *aggregate* indemnity reimbursed to the policyholders, and hence the premium the latter are asked to pay, are not fixed; both depend on the realization of K. This implies that the premium is *random*.

Definition 1 *The mutual agreement is such that the aggregate amount of indemnities to be paid to policyholders belonging to the pool ($KqL(e)$) is equally shared among the n members of the pool. Thus, the individual premium is: $\frac{KqL(e)}{n}$.*[6]

Importantly, the total amount of premiums collected in the pool exactly covers the total amount of indemnities paid to the K individuals experiencing loss. This is a standard property of mutual insurance: profits are always zero ex-post.[7]

The timing of actions is the following:

- the percentage of the loss to be reimbursed to policyholders, q, is chosen cooperatively in the pool.
- the policyholders choose the effort level.
- the risk (and thus the number of individuals experiencing loss K) is realized.

3.1 Non-cooperative strategy in mutual insurance

We will show that, in the second stage, policyholders' choice of effort can be either cooperative or not. The non-cooperative standpoint has been already considered in the case of ex-ante moral hazard (see Lee and Ligon 2001). In this subsection and in Section 4 we consider the non-cooperative strategy in the case of a self-insurance effort.

If she uses a non-cooperative strategy, the representative consumer only internalizes the effect *of her own effort* on the random premium in the event she herself experiences the loss. In other words, the consumer neglects the "social benefit" of the effort *on the aggregate loss* in both states of nature. Thus, given that k consumers out of the others $(n-1)$ in the pool experience a loss, the representative consumer i's expected utility is:

$$EU_i^{NC}(k) = pU\left(w - \frac{q}{n}\left(L(e_i) + kL(e_{-i})\right) - L(e_i) + qL(e_i)\right) \\ + (1-p)\,U\left(w - \frac{kq}{n}L(e_{-i})\right) - C(e_i) \tag{8}$$

where e_i is the effort exerted by consumer i and e_{-i} is the effort exerted by the other $n-1$ consumers in the pool.

[6] Note that such an "equal sharing rule" is not the optimal incentive-compatible rule. The optimal rule would be obtained by maximizing the expected utility of a representative policyholder under the incentive constraint, and a resource constraint that would have to be fulfilled in every state of nature (and not only in expectation, as in the case of a stock insurer). Still, the equal sharing rule defined above seems to better describe actual mutual (in particular, health) insurance contracts.

[7] If at the end of the period the aggregated indemnities to be reimbursed by the mutual are greater than the premiums collected, consumers are asked to pay an additional premium. If, on the contrary, the aggregated indemnities to be reimbursed by the mutual are lower than the premiums collected, consumers should receive back a part of the premium paid. In the latter case, in reality consumers rarely receive money back ; the mutual insurance generally prefers to use "profits" to increase its contingency reserves and funds.

Note that here we are considering two different cases regarding the total number of consumers that experience a loss (K). As we fix k (that is the number of consumers that experience a loss out of the $(n-1)$ others), if the representative policyholder herself experiences a loss then $K = k+1$ whereas, if she doesn't, $K = k$. Moreover, it appears that consumer i behaves as if the impact of her own effort on the premium were different in the two preceding cases. In particular, if the policyholder suffers the loss, she perceives that her premium also depends on her effort, such that the premium writes $\frac{q}{n}\left(L(e_i) + kL(e_{-i})\right)$. Whereas, if she does not suffer the loss, she perceives that her premium does not depend on her effort, such that the premium writes $\frac{kq}{n}L(e_{-i})$.

To summarize, in the non-cooperative case each policyholder only internalizes part of the effect of her own effort on the premium, i.e. $\frac{q}{n}L(e_i)$, and only in the event that the loss occurs.

3.2 Cooperative strategy in mutual insurance

We innovate with respect to the existing literature on mutual insurance by proposing the opportunity to choose a cooperative effort strategy within a mutual arrangement, thus making it possible to fully internalize the "social benefit" of the policyholders' effort on the aggregate loss.

Definition 2 *Cooperation is the situation where all policyholders agree on a common level of effort that maximizes the expected welfare of a representative policyholder.*

Therefore, we consider here the representative consumer's problem as a function of her choice of cooperative effort when all the other policyholders cooperate as well. In the cooperative case, the representative consumer's expected utility given that k members out if the others $(n-1)$ experience a loss is:

$$EU(k) = pU\left(w - L(e) - \frac{(k+1)qL(e)}{n} + qL(e)\right) + (1-p)\,U\left(w - \frac{kqL(e)}{n}\right) - C(e) \quad (9)$$

where e is the effort exerted by all the cooperating consumers in the pool. Therefore, under cooperation, all members take into account the "social benefit" of the effort on the aggregate loss.

In the following sections, we will first analyze the non-cooperative game, and then explain the policyholders' payoff under cooperation. Finally, in Section 6, we will show how policyholders' welfare changes with size of pool both in the non-cooperative equilibrium and under cooperation, and we will identify conditions such that cooperation can be sustained as an equilibrium.

4 Non-cooperative equilibrium in mutual insurance

When policyholders do not cooperate, the representative consumer i's expected utility given k is expressed in (8) above.

Let us call $b(k; n-1; p)$ the binomial probability of k losses with $n-1$ individuals with probability of loss p. As previously explained, net consumption is $W_L^{NC} = w - \frac{q}{n}\left(L(e_i) + kL(e_{-i})\right) - L(e_i) + qL(e_i)$ when individual i experiences the loss and $W_0^{NC} = w - \frac{kq}{n}L(e_{-i})$ when she does not. The representative consumer's expected utility is therefore:

$$EU_i^{NC} = \sum_{k=0}^{n-1} b(k; n-1; p)\{pU\left(W_L^{NC}\right) + (1-p)\,U\left(W_0^{NC}\right)\} - C(e_i) \quad (10)$$

Solving backward, in the second step the representative consumer chooses her own effort. The optimal effort level is:

$$\arg\max_{e_i} EU^{NC}(e_i; q, e_{-i})$$

In particular:

$$e_i^{*NC}(n, q, e_{-i}) : \tag{11}$$
$$\sum_{k=0}^{n-1} b(k; n-1; p)\{pU'\left[W_L^{NC}\right](-(1-q)L'(e_i) - \frac{q}{n}L'(e_i))\} = C'(e_i)$$

Interestingly, from (11) and contrary to (6), we see that under a mutual agreement the effort is positive even in the case of full coverage ($q = 1$) since the marginal benefit of the effort is always greater than zero. The reason is that here policyholders internalize part of the beneficial impact of the effort on the premium through the term $\frac{q}{n}L(e_i)$ appearing in the l.h.s. of equation (11).

With identical agents, the equilibrium is symmetric and $e_i = e_{-i} = e$. So that, by expressing the binomial distribution:

$$e^{*NC}(n, q) : \tag{12}$$
$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{k!(n-k-1)!} p^k (1-p)^{n-k-1} \{pU'\left[W_L^{NC}\right](-L'(e)(1-q+\frac{q}{n})\}$$
$$= C'(e)$$

We can now consider the first step of the game: since they always act cooperatively in the first stage, the mutual insurance policyholders choose the optimal coverage as follows:

$$\begin{aligned} &\max_q \textstyle\sum_{k=0}^{n-1} \tfrac{(n-1)!}{k!(n-k-1)!} p^k (1-p)^{n-k-1} \left\{pU\left[W_L^{NC}\right] + (1-p)\,U\left[W_0^{NC}\right]\right\} - C(e) \\ &s.t. : \textstyle\sum_{k=0}^{n-1} \tfrac{(n-1)!}{k!(n-k-1)!} p^k (1-p)^{n-k-1} \{pU'(\left[W_L^{NC}\right](-L'(e)(1-q+\tfrac{q}{n})\} = C'(e) \end{aligned} \tag{13}$$

Where $W_L^{NC} = w - \frac{(k+1)q}{n}L(e) - (1-q)L(e)$, $W_0^{NC} = w - \frac{kq}{n}L(e)$ and, $e \equiv e^{*NC}$.

Lemma 1 *When a self-insurance measure is available and policyholders act non-cooperatively in the second stage, the mutual agreement offers full coverage* $\left(q^{*NC} = 1\right)$.

Proof. See the Appendix 9.1. ■

From (11) recall that the optimal choice of effort is positive even when $q^{*NC} = 1$. Thus, Lemma 1 shows that a non-cooperative strategy among individuals in the choice of effort leads to full coverage for a given K and to positive effort $e^{*NC}(n, 1)$. However, as the premium depends on K (see Definition 1), full coverage does not lead to full insurance: $W_L^{*NC} = w - \frac{k+1}{n}L(e^{*NC}) < W_0^{*NC} = w - \frac{k}{n}L(e^{*NC})$.

Note that this full coverage contrasts with the second-best contract (see equation 6). Full coverage is optimal in mutual insurance since policyholders still face the risk related to the random premium, so that they retain the incentive for a positive effort. In particular:

$$e^{*NC}(n, 1) : \tag{14}$$
$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{k!(n-k-1)!} p^k (1-p)^{n-k-1} \left[pU'\left(w - \frac{k+1}{n}L(e^{*NC})\right)\left(-\frac{1}{n}L'(e^{*NC})\right)\right]$$
$$= C'(e^{*NC})$$

Substituting non-cooperative effort in the consumers' expected utility, the policyholder's payoff in the non-cooperative equilibrium is:

$$EU^{NC} = \sum_{k=0}^{n-1} \tfrac{(n-1)!}{k!(n-k-1)!} p^k (1-p)^{n-k-1} \left\{ pU\left(w - \tfrac{(k+1)L(e^{*NC})}{n}\right) \right. \tag{15}$$
$$\left. +(1-p)U\left(w - \tfrac{kL(e^{*NC})}{n}\right)\right\} - C(e^{*NC})$$

Note again that, even with full coverage $\left(q^{*NC}=1\right)$, the expected utility of the representative policyholder depends on her own realization of the loss, as the latter changes the total number of losses K (since $K = k+1$ if she experiences a loss and $K = k$ otherwise).

5 Cooperation in mutual insurance

Under cooperation, the representative consumer i's expected utility given k is expressed in (9) above.

Considering the binomial function, expected utility is:

$$EU^C = \sum_{k=0}^{n-1} b(k; n-1; p)\{pU\left(W_L^C\right) + (1-p)\,U\left(W_0^C\right)\} - C(e) \tag{16}$$

where, under cooperation (see discussion in Subsection 3.2), net consumption can be written as: $W_L^C = w - \frac{(k+1)q}{n}L(e) - L(e) + qL(e)$ and $W_0^C = w - \frac{kq}{n}L(e)$

The optimal choice of effort given the risk-sharing rule and coverage q, here is:

$$\arg\max_e EU^C(e; n, q)$$

In particular:

$$e^{*C}(n,q):$$
$$\sum_{k=0}^{n-1} b(k; n-1; p)\Big\{p(U'\left[W_L^C\right](-L'(e) + \tfrac{(n-(k+1))}{n}qL'(e))$$
$$+(1-p)U'\left[W_0^C\right]\left(-\tfrac{kqL'(e)}{n}\right)\Big\} = C'(e)$$

Rearranging and making the binomial distribution explicit we have:

$$e^{*C}(n,q): \tag{17}$$
$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{k!(n-k-1)!} p^k (1-p)^{n-k} \Big\{ -\left(1 - \tfrac{n-1}{n}q\right) L'(e)pU'(W_L^C)$$
$$-E\left[U'(W^C)\right] L'(e)\tfrac{k}{n}q\Big\} = C'(e)$$

where $E\left[U'(W^C)\right] = pU'\left(W_L^C\right) + (1-p)\,U'\left(W_0^C\right)$.

We can now consider the first step of the game: since they always act cooperatively in the first stage, mutual insurance policyholders solve:

$$\max_{q}\sum_{k=0}^{n-1}\frac{(n-1)!}{k!(n-k-1)!}p^{k}(1-p)^{n-k-1}\{pU\left(W_{L}^{C}\right)+(1-p)\,U\left(W_{0}^{C}\right)\}-C(e)$$

$$s.t.:\sum_{k=0}^{n-1}\tfrac{(n-1)!}{k!(n-k-1)!}p^{k}(1-p)^{n-k}\left\{-\left(1-\frac{n-1}{n}q\right)L'(e)pU'(W_{L}^{C})\right.$$

$$\left.-E\left[U'(W^{C})\right]L'(e)\frac{k}{n}q\right\}=C'(e) \tag{18}$$

Lemma 2 *When a self-insurance measure is available and policyholders act cooperatively in the second stage, the mutual agreement offers full coverage* $\left(q^{*C}=1\right)$.

Proof. The proof is very similar that of Lemma 1 and is thus omitted. ■

From (17) recall that the optimal choice of effort is positive when $q^{*C}=1$. In particular, from Lemma 2, the optimal effort becomes:

$$e^{*C}(n,1): \tag{19}$$

$$\sum_{k=0}^{n-1}\tfrac{(n-1)!}{k!(n-k-1)!}p^{k}(1-p)^{n-k-1}\Big[pU'\left(w-\frac{k+1}{n}L(e)\right)\left(-\frac{k+1}{n}L'(e)\right)$$

$$+(1-p)U'\left(w-\frac{k}{n}L(e)\right)\left(-\frac{k}{n}L'(e)\right)\Big]=C'(e)$$

or

$$\sum_{k=0}^{n-1}\tfrac{(n-1)!}{k!(n-k-1)!}p^{k}(1-p)^{n-k}\Big\{-\left(\tfrac{1}{n}\right)L'(e)pU'\left(w-\frac{k+1}{n}L(e)\right)$$

$$-E\left[U'(W^{C})\right]L'(e)\tfrac{k}{n}\Big\}=C'(e)$$

Comparing equations (6) and (14) with equation (19), we note that the new term $-E\left[U'(W^{C})\right]L'(e)\frac{k}{n}>0$ appears in the l.h.s. of the latter, that is in the marginal benefit of effort. Such a term represents the positive impact that a higher effort has on the random premium and, thus, on marginal utility of net consumption in both states of the world. Finally, as we expected, from (14) and (19) it follows that $e^{*NC}(n,1)<e^{*C}(n,1)$.

The policyholder's payoff under cooperation can be expressed as:

$$EU^{C}=\sum_{k=0}^{n-1}\tfrac{(n-1)!}{k!(n-k-1)!}p^{k}(1-p)^{n-k-1}\left\{pU\left(w-\tfrac{(k+1)L(e^{*C})}{n}\right)\right. \tag{20}$$

$$\left.+(1-p)U\left(w-\tfrac{kL(e^{*C})}{n}\right)\right\}-C(e^{*C})$$

Recall that the policyholder's payoff in the non-cooperative equilibrium is expressed in (15) above. As is easily verified, the sole difference between (15) and (20) lies in $e^{*NC}<e^{*C}$.

Summarizing results in the last two sections:

Corollary 1 *Both in the non-cooperative equilibrium and under cooperation, the optimal coverage in mutual insurance is full coverage. However policyholders exert a higher level of effort under cooperation than in the non-cooperative equilibrium.*

We will prove in the next section that, as we expect, policyholders always receive a higher utility under cooperation than in the non-cooperative equilibrium.

6 Pool size and efficiency in mutual insurance

In this section we first investigate how policyholders' payoff changes with size of pool both in the non-cooperative equilibrium and under cooperation. Then we show that cooperation can be sustained as an equilibrium within a repeated interaction game with punishment strategies.

First of all consider that, when $n = 1$, the non-cooperative payoff and the cooperative payoff are obviously identical and represent a situation where the representative consumer is not insured:

$$EU^1 = pU\left[w - L(e)\right] + (1-p)\, U\left(w\right) - C(e)$$

The FOC with respect to the effort in this case is:

$$e^{*1} : -pU'\left[w - L(e)\right] L'(e) = C'(e)$$

Let us now consider how the non-cooperative equilibrium payoff and the outcome under cooperation both change with pool size:

Lemma 3 *Both in the non-cooperative equilibrium and in the cooperation outcome, consumers' expected utility is monotonically increasing with size of pool.*

Proof. See the Appendix 9.2. ■

Note that the previous result is an extension of the Borch (1962) rule to the case where consumers choose a self-insurance measure: the benefit of risk-sharing increases with size of pool.

Having proved that the policyholders' payoff is monotonically increasing with size of pool both in the non-cooperative equilibrium and under cooperation, in the next subsection we show that the shape of the two policyholders' payoffs is different, such that expected utility under cooperation always dominates expected utility in the non-cooperative equilibrium.

6.1 Asymptotic results

Recall that K is the total number of individuals in the pool experiencing the loss. As the size of the pool tends towards infinity, the share of individuals suffering the loss $\left(\frac{K}{n}\right)$ tends towards the probability of the loss p:

Remark 1 *According to the law of large numbers, as $n \to \infty$, we obtain that:*

$$\frac{K}{n} \to p$$

This result implies that, with a very large pool, the random premium in the mutual company $\frac{KqL(e)}{n}$ tends towards the fixed premium $P = pqL(e)$ of a stock insurance company (see subsection 2.3 before), thus nullifying premium uncertainty in the risk-pooling. This occurs both in the non-cooperative equilibrium and under cooperation. Considering that in the first stage the mutual insurance policyholders optimally choose $q = 1$:

Corollary 2 *When the number of individuals insured in the mutual company is infinite, the random premium converges to the fixed (independent of K) premium $pL(e)$ both in the non-cooperative equilibrium and under cooperation.*

Let us consider the first-order condition for effort in the non-cooperative equilibrium (14). Applying Remark 1, as $n \to \infty$ the first-order condition becomes:

$$pU'(w - pL(e)) \left(-\frac{1}{n}L'(e)\right) = C'(e)$$

or:

$$e_{\infty}^{*NC} = 0 \tag{21}$$

Thus, the consumers' payoff , as $n \to \infty$, in the non-cooperative equilibrium is:

$$EU_{NC}^{\infty} = U(w - pL(0)) - C(0) = U(w - pL(0))$$

From (21), when the number of policyholders in the pool goes to infinity, the consumers' incentive to exert a positive effort disappears since the marginal benefit of the effort becomes zero. This implies that policyholders' expected utility is characterized by full insurance, the highest loss and no disutility from the effort.

Let us now consider the first-order condition for the cooperative effort (19). Applying Remark 1, as $n \to \infty$ the first-order condition becomes:

$$pU'\left(w - pL(e^{*C})\right)\left(-pL'(e^{*C})\right) + (1-p)U'\left(w - pL(e^{*C})\right)\left(-pL'(e^{*C})\right) = C'(e^{*C})$$

or:

$$e_{\infty}^{*C} : U'(w - pL(e))\left(-pL'(e)\right) = C'(e) \tag{22}$$

By comparing (3) and (22) we can easily verify that:

$$e_{\infty}^{*C} \equiv e^{FB}$$

Thus, the consumers' payoff under cooperation when $n \to \infty$ replicates the first-best:

$$EU_{\infty}^{C} = U(w - pL(e^{FB})) - C(e^{FB})$$

The previous reasoning is stated in Lemma 4:

Lemma 4 *When the number of individuals in the pool is infinite, cooperation in mutual insurance implements the first-best allocation.*

Recall that, by definition, e^{*C} =arg $\max_{e} EU(e; e)$ and $e^{*NC} = \arg\max_{e_i} EU(e_i; e^{*NC})$. Since, the cooperative effort maximizes the individual's expected utility when all members in the pool choose the same effort level, the non-cooperative equilibrium, which is symmetric, is necessarily dominated by the cooperative outcome. We can state the following:

Remark 2 *The cooperative outcome always dominates the non-cooperative equilibrium.*

Figure 1 below shows a graphical representation of Remark 2.

6.2 Cooperation as an equilibrium

We now investigate conditions under which cooperation in mutual agreements can be an equilibrium.

It can easily be shown that cooperation in the second stage of the game considered in Section 3 can never be an equilibrium. Suppose that the optimal choice of q has already been taken by the mutual in the previous stage ($q^{*C} = 1$). Under full coverage, the policyholder's best response e_i^{BR}, when all the other policyholders choose the cooperative strategy e^{*C}, is obtained from (10) as follows:

$$\max_{e_i^{BR}} EU_i = \sum_{k=0}^{n-1} b(k; n-1; p)\Big\{pU\left[w - \frac{1}{n}\left(L(e_i^{BR}) + kL(e^{*C})\right)\right] + (1-p)\,U\left[w - \frac{k}{n}L(e^{*C})\right]\Big\} - C(e_i^{BR}) \tag{23}$$

Thus, the best response to the cooperative effort e^{*C} is:

$$e_i^{BR}(n,1): \tag{24}$$

$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{k!(n-k-1)!} p^k (1-p)^{n-k-1} \left\{pU'\left[w - \tfrac{1}{n}\left(L(e_i^{BR}) + kL(e^{*C})\right)\right]\left(-\tfrac{1}{n}L'(e_i^{BR})\right)\right\} = C'(e_i^{BR})$$

which is lower than the cooperative effort, i.e.:

$$e_i^{BR} < e^{*C}$$

The preceding result is not surprising: the costly effort by each policyholder in the pool exerts a positive externality on the random premium and the policyholder prefers to *free ride* on it. The most advantageous situation for a member of the pool is when all the other policyholders internalize the social benefit of the effort on the random premium, while she alone internalizes the impact of her effort on her own loss in the event it occurs. As is always the case for positive externalities, the market (or non-cooperative) solution implies *underprovision of the effort.* In our context this implies that cooperation cannot be sustained as an equilibrium in the one-shot interaction among policyholders $\left(e_i^{BR} \neq e^{*C}\right)$.

In the following we will consider how the mutual insurance scheme can restore efficiency by implementing the cooperative solution. We will investigate a case where the members of the pool interact for an uncertain number of periods. We will see that cooperation can be sustained as an equilibrium if the members of the pool implement a punishment when they observe a loss which is higher than expected. A high loss means that someone deviated from the cooperative effort by exerting a low effort. As a consequence, to punish the deviator, all other policyholders exert the lowest effort in all the subsequent periods.

In particular, we adapt the Folk Theorem to our environment by interpreting the time horizon as uncertain. Note that deviation by one policyholder is detected by the other members of the pool *only if* the deviator experiences the loss, since the size of her loss is higher than expected in this case. Thus, we are in a stochastic environment regarding deviation observability.[8]

Let us call δ the discount factor. We apply the Grim trigger strategy in our context: in the first period $t = 0$ the policyholder chooses the cooperative effort e^{*C}. In each following period $t > 0$ the

[8]The identity of the deviator is not necessarily known by the other policyholders.

policyholder exerts e^{*C} if all policyholders have chosen e^{*C} in each past period $t-1$, otherwise she exerts $e=0$ forever.

Let us call $EU(e^{*C}, e^{*C}, n)$ the policyholder's payoff when she cooperates and $EU(e_i^{BR}, e^{*C}, n)$ her payoff when she deviates (see expression (23)). In the period after deviation, the policyholder will be detected only if the loss is realized, that is with probability p. In that case, all the other members of the pool will punish her by choosing the lowest effort forever. Let us call $EU(0,0,n)$ the payoff the deviator obtains when she is detected[9]. With probability $(1-p)$ her deviation is not detected and the policyholder obtains the payoff $EU(e_i^{BR}, e^{*C}, n)$ even in the period after the deviation[10], and in the same way in the subsequent periods. After a deviation the policyholder's payoff can be written as follows:

$$\begin{aligned}
&EU(e_i^{BR}, e^{*C}, n) + \delta p\left(\frac{1}{1-\delta}EU(0,0,n)\right) + \\
&\delta(1-p)\left\{EU(e_i^{BR}, e^{*C}, n) + \delta p\frac{1}{1-\delta}EU(0,0,n) + \right. \\
&\left.\delta(1-p)\left[EU(e_i^{BR}, e^{*C}, n) + \delta p\frac{1}{1-\delta}EU(0,0,n) + \delta(1-p)[...]\right]\right\}
\end{aligned}$$

Thus, the discounted payoff in the case of deviation is:

$$\frac{\delta p}{1-\delta}\sum_{t=0}^{\infty}\left(\delta\left(1-p\right)\right)^t EU(0,0,n) + \sum_{t=0}^{\infty}\left(\delta\left(1-p\right)\right)^t EU(e_i^{BR}, e^{*C}, n)$$

or:

$$\frac{1}{1-\delta\left(1-p\right)}\left[EU(e_i^{BR}, e^{*C}, n) + \frac{p\delta}{1-\delta}EU(0,0,n)\right]$$

Whereas, if the policyholder cooperates forever, the discounted payoff she obtains is:

$$\sum_{t=0}^{\infty}\delta^t EU(e^{*C}, e^{*C}, n) = \frac{1}{1-\delta}EU(e^{*C}, e^{*C}, n).$$

From the previous reasoning, cooperation can be sustained as an equilibrium if the following inequality holds:

$$\frac{1}{1-\delta}EU(e^{*C}, e^{*C}, n) \geq \frac{1}{1-\delta\left(1-p\right)}\left[EU(e_i^{BR}, e^{*C}, n) + \frac{p\delta}{1-\delta}EU(0,0,n)\right] \tag{25}$$

[9]The strategy $e=0$ for the deviator can be interpreted as the subgame perfect Grim trigger strategy.

We do not claim here that the Grim trigger strategy is optimal in our context. We use it to make our problem more tractable. If a more efficient punishment strategy exists, this is good news, since it implies that competition can be more easily sustained as an equilibrium in our setting.

[10]The strategy $e = e^{BR}$ for the deviator results from the Bellman principle:

$$\begin{aligned}
V_0 &= \max_e \left\{EU(e, e^{*C}, n) + \delta\left[pV_N + (1-p)\,V_0\right]\right\} \\
&= EU(e^{BR}, e^{*C}, n) + \delta\left[pV_N + (1-p)\,V_0\right]
\end{aligned}$$

where V_0 is the present value of the policyholder's best-reply when her deviation is not detected, whereas V_N is the present value of the policyholder best response when her deviation is detected.

Proposition 1 *There is a threshold value $\hat{n}$ such that cooperation is enforceable for $\forall n \leq \hat{n}$.*

Proof. See the Appendix 9.3. ■

The previous proposition states that, as we expected, only a sufficiently low pool size is compatible with the cooperative equilibrium. In Subsection 7.2 below we describe simulations where cooperation is sustained for given values of n and γ and for sufficiently high δ.

7 Comparing mutual and stock insurance policies

We now compare consumers' expected utility in the mutual agreement with consumers' expected utility under second-best stock-type contract.

7.1 Analytical results

Let us consider the contract illustrated in Subsection 2.3. Such a policy provides partial insurance at a fixed premium and is referred to as the second-best contract since it always implies a lower consumers' welfare than the first-best contract. The second-best policy is independent of the number of individuals insured by the firm: whatever the number of policyholders, the representative consumer always obtains the same contract (P, q) , with $q < 1$, $P = pqL(e)$, and she chooses the effort $e^{*SB}(q)$ such that: $-(1-q)L'(e)pU'(W_L) = C'(e)$.

Remark 3 *The second-best contract always dominates the non-cooperative equilibrium outcome in mutual insurance.*

Proof. Obviously for $n = 1$ the second-best contract dominates the non-cooperative equilibrium outcome in mutual insurance (which, for $n = 1$, corresponds to no-insurance). The consumer's welfare under the second-best contract is independent of n, whereas from Lemma 3 we know that the non-cooperative payoff is monotonically increasing with size of pool. Finally, for $n \to \infty$, the non-cooperative payoff becomes $EU_{NC}^{\infty} = U(w - pL(0))$. This payoff is necessarily lower than consumers' welfare under the second-best contract since the latter contract just *maximizes* consumers' welfare and obtains partial coverage ensuring a positive effort (the *possible* solution $q = 1$ and $e = 0$ is not the optimal one). ■

Note that Remark 3 replicates Ligon and Thistle (2008)'s result in cases where a self-insurance measure is available to policyholders. In particular, Ligon and Thistle showed that, in the case of *ex-ante* moral hazard and non-cooperative choice of effort, consumers' welfare is higher when individuals purchase an insurance policy on a competitive stock insurance market than when they purchase a policy from a mutual insurer.

From Lemmas 3 and 4 we can state the following:

Proposition 2 *When the number of policyholders in the pool is sufficiently large, a mutual policy under cooperation dominates the second-best contract.*

Interestingly, Proposition 2 implies that there is a threshold value n^* at which a mutual policy under cooperation generates the same welfare as the second-best contract. Obviously, for $n < n^*$ a mutual policy under cooperation is dominated by the second-best contract.

Recall that Proposition 1 showed that cooperation can be sustained as an equilibrium for a sufficiently low size of pool ($n \leq \hat{n}$). We need now to consider whether $\hat{n} \geq n^*$, that is whether cooperation in the mutual can be sustained as an equilibrium for pool size values such that the mutual dominates the second-best contract. Only if $\hat{n} \geq n^*$ can we claim that mutual insurance beats the second-best contract.[11] We will show that this previous inequality is easily verified with some simulations in the next subsection. With reasonable functions describing policyholders' utility (using CARA, CRRA and quadratic functions), we find that the cooperative equilibrium dominates the second-best contract even for low pool sizes. For example $\hat{n} \geq n^* = 4$ for the CRRA utility function with risk aversion parameter $\gamma = 0.7$ and $\hat{n} \geq n^* = 9$ for the same function with $\gamma = 1.2$, suggesting that the benefits from risk-sharing and cooperation are sufficiently high even for small-sized mutual arrangements.

By proving that mutual insurance can sustain cooperation among its members and that cooperation leads to the first-best allocation for $n \to \infty$, we extend Ligon and Thistle (2008)'s result to the case of cooperation in the choice of effort and show that a mutual scheme of a sufficiently large size can dominate a second-best policy provided by a stock insurer.

All results described in this section are illustrated in Figure 1 below.

7.2 Simulations

In this section we use simulations to show that the threshold value n^* sufficient to ensure that the mutual agreement under cooperation dominates the second-best insurance policy (see Proposition 2) is indeed a fairly low value. Moreover, for such a size of pool we show that cooperation is sustainable as an equilibrium in the repeated interaction game for a plausible value of the discount factor. This implies that in all simulations performed $n^* < \hat{n}$ (see Proposition 1).

For our simulations, we use the most common utility functions: CRRA (constant relative risk aversion), CARA (Constant absolute risk aversion) and some polynomials. In the tables below we report the results for the CRRA function:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

where γ represents the degree of relative risk-aversion. Very similar results were obtained for the other utility functions and are available from the authors on request.

We present in this section the results for two parametric values for the degree of relative risk aversion: $\gamma = 0.7$ and $\gamma = 1.2$. The first is the mean value of the degree of relative risk aversion estimated by Chetty (2006) using data on labor supply behavior; the second comes from Meyer and Meyer (2005) who find that relative risk aversion with respect to wealth is near, but greater than, one.

[11] It could be objected that repeated interactions in the mutual agreement provide a large (unfair) advantage to the mutual insurance. Note, however, that there is no point in considering repeated interactions (and their possible advantages for the insurance firm) in the case of a competitive market where second-best contracts are offered. To understand why, suppose that at the end of the period a higher than expected loss is observed, so that it is inferred that a policyholder exerted a low effort. The insurer cannot in that case punish such a policyholder in the subsequent period (for example by imposing a higher premium) since the policyholder will purchase a contract from a rival insurance company at the market competitive price $P = pqL(e^{*SB})$.

Wealth w is 2 and the value for the probability of loss p is 0.3. The cost of the effort is expressed by the function $C(e) = e^2$, while the loss is $L(e) = 1 - e$.

	γ= 0.7			
Contract policies	n	q	e	EU
First-best	$-$	1	0.102175	3.9191
Second-best contract	$-$	0.69927	0.033855	3.91177
Mutual policy (cooperation)	4	1	0.109917	3.91306
	4500	1	0.102181	3.9191
Mutual policy (non-cooperation)	4	1	0.013899	3.9036
	4500	1	0.00001284	3.90852

(Table 1a)

The simulation results show that, given a degree of risk aversion $\gamma = 0.7$, a low pool size ($n = 4$) is sufficient for a mutual scheme with cooperation to dominate the second-best stock-type insurance policy. Further, as we expected, the mutual policy under cooperation and the second-best contract always dominate the non-cooperative mutual contract whatever the size (in the table values are given as $n = 4$ and $n = 4500$). Finally for $n = 4500$, mutual insurance in cooperation replicates the first-best outcome.

Note that, for both mutual policies, as pool size increases policyholders' effort decreases. The same phenomenon can be observed in Table 1b below for relative risk aversion $\gamma = 1.2$. To intuit this result, the reader can consider the l.h.s. of FOCs (14) and (19). In both FOCs the marginal benefit of effort decreases with size of pool, so that the higher the number of policyholders, the lower the incentives for the effort.

When the degree of risk aversion rises to $\gamma = 1.2$, the pool size required for the mutual under cooperation to dominate the second-best contract increases to at least 9 (see table below) [12]. Intuitively, the reason for this result is that the effect of risk aversion on pool size is connected with *macroscopic* risk. A higher degree of risk aversion directly indicates a lower preference for the mutual policy because of aggregate risk. In other words, the more risk-averse the consumer is, the less she likes mutuals, since they make her support the aggregate risk. Thus, mutual policies need to be larger (favoring risk sharing, see Lemma 3 above) to imply a lower macroscopic risk and consequently to dominate the second-best contract.

	γ= 1.2			
Contract policies	n	q	e	EU
First-best	$-$	1	0.078058	-4.49036
Second-best contract	$-$	0.8443	0.013319	-4.49557
Mutual policy (cooperation)	9	1	0.083377	-4.49451
	7500	1	0.078061	-4.49036
Mutual policy (non-cooperation)	9	1	0.004331	-4.50093
	7500	1	0.0000317	-4.49656

(Table 1b)

Observation 1 *For the set of specifications chosen, the higher the consumer's risk aversion, the higher the pool size required for the mutual cooperative policy to dominate the second-best contract.*

[12]Note that, under CRRA preferences, the expected utility is negative for $\gamma > 1$.

A rise in the degree of risk aversion also entails a general reduction in the effort exerted by the representative policyholder. This second observation is analytically supported by the following remark.

Remark 4 - *With CRRA preferences, in both the cooperative and the non-cooperative case, the optimal levels of effort are decreasing with the coefficient of relative risk aversion when* $L^{''}(e) \geq 0$ *and* $w - L(0) > 1$.

- *Similarly, with CARA preferences* $\left(U(W) = -\frac{1}{\alpha}e^{-\alpha W}\right)$, *in both the cooperative and the non-cooperative case, the optimal levels of effort are decreasing with the coefficient of absolute risk aversion when* $L^{''}(e) \geq 0$.

Proof. See the Appendix 9.4. ■

A second important result from our simulations concerns the sustainability of the cooperative solution as an equilibrium. For the low pool size values $n = 4$ (when γ= 0.7) and $n = 9$ (when γ= 1.2) and with the discount factor at values $\delta \geq \hat{\delta} = 0.58$ and $\delta \geq \hat{\delta} = 0.72$ respectively, cooperation can be sustained as an equilibrium (see equation 25).[13] Therefore the following inequalities hold: $n^* = 4 < \hat{n}$ (for γ= 0.7 and for $\delta \geq \hat{\delta} = 0.58$) and $n^* = 9 < \hat{n}$ (for γ= 1.2 and for $\delta \geq \hat{\delta} = 0.72$). In other words: considering the degree of risk aversion $\gamma = 0.7$ (respectively γ= 1.2), for $n = 4$ (respectively $n = 9$) cooperation can be sustained as an equilibrium and a mutual policy with cooperation dominates the second-best stock-like insurance policy. This is illustrated in Table 2. As can easily be verified, values in the table below verify equation (25).

policyholder's payoff	n	Effort	Payoff
cooperation: $EU(e^{*C}, e^{*C}, n)$	4	0.109917	3.91306
deviation: $EU(e_i^{BR}, e^{*C}, n)$	4	0.027739	3.91984
punishment: $EU(0, 0, n)$	4	0	3.90066

(Table 2)

Finally, we propose a figure which perfectly describes the main results of the paper.

[13]The value $\hat{\delta} = 0.72$ is plausible in a mutual agreement, as discussed in the concluding section.

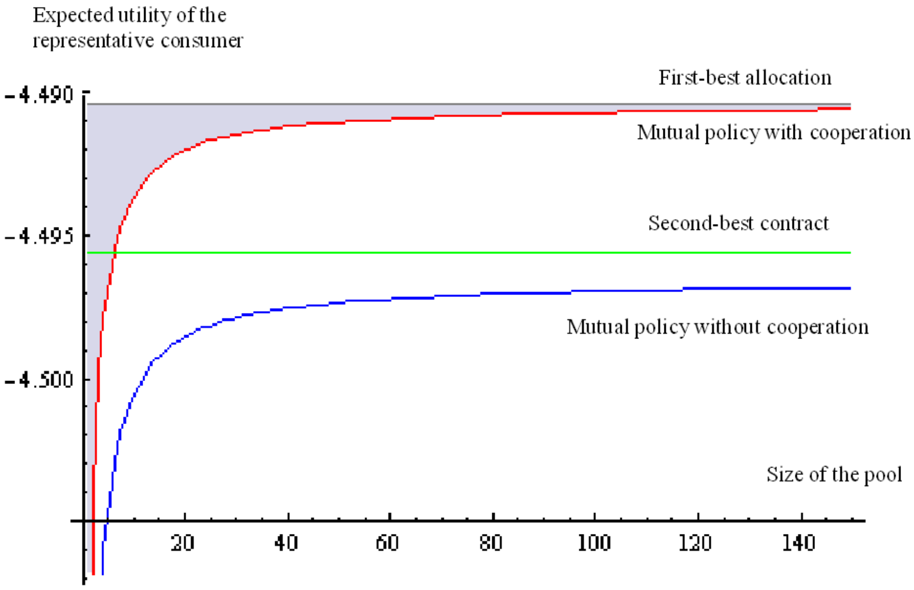

Figure 1: First-best, second-best, and mutual policies with and without cooperation for CRRA preferences with a coefficient of relative risk-aversion of 1.2

8 Concluding remarks

Our focus here is on mutual insurance policies where the consumer can exert a self-insurance effort that decreases the amount of any loss occurring. The policyholder decides how much effort to exert before the risk is realized. We analyze both the case where the policyholder chooses the effort non-cooperatively and the case where she chooses the effort in cooperation with the other members of the pool. Differences between the non-cooperative and cooperative strategies depend on the (partial or whole) internalization of the impact that individuals' effort has on the random premium. We investigate cooperative strategies in particular, since they lead to full internalization of the positive externality.

As intuition suggests, we find that the mutual policy under cooperation implies a higher effort and always dominates the non-cooperative mutual policy in terms of policyholders' expected utility. We also show that, when pool size is infinite, mutual arrangements under cooperation replicate the first-best allocation. Moreover, with repeated interactions, cooperation can be sustained as an equilibrium when there is a sufficiently low pool size (and for a sufficiently high policyholders' discount factor).

An interesting consequence of the asymptotic result concerning mutual arrangements is that, provided the number of policyholders in the pool is sufficiently large, a cooperative mutual policy dominates a second-best stock-like insurance contract. Importantly, our numerical simulations show that the size of pool required for a cooperative mutual policy to dominate a second-best stock-type insurance contract is fairly low, and this threshold guarantees the tenability of cooperation as an equilibrium.

Our results thus suggest that mutual agreements under cooperation can beat stock insurance policies. This is more likely when risk aversion is low, the discount factor is high and the size of the pool is neither too high nor too low. In other words, risk-sharing in the mutual under cooperation can be so effective that the negative impact of the macroscopic risk is almost nullified even for a fairly small pool size. Our findings extend previous results which showed that a second-best stock insurance contract always dominates a mutual policy under ex-ante moral hazard and choice of non-cooperative effort (Ligon and Thiestle 2008).

The cooperative behavior we consider does not require an individual to be empathetic with other members within the pool, coming from an absolutely standard utility maximizer attitude. However, supportive, fair and conditional cooperative behaviors can be considered plausible in a mutual arrangement, given the very specific nature of the participating contract. In other words, willingness to cooperate may be higher in individuals who self-select in mutual arrangements. Solidarity principles explicitly mentioned in the mutual insurance articles of association/incorporation, if accepted by the policyholders, could well provide a better route to cooperation.[14] Moreover, deviation can also entail psychological costs to the deviator in terms of, for example, lowered self-esteem or a social stigma. Finally, although constrained in the standard context of purely selfish preferences, members of the mutual agreement partially know each other since they assemble for periodical meetings. This suggests that some partial peer monitoring is possible when consumers purchase a mutual policy.[15]

[14] The beneficial matching between agents characterized by a similar "mission" (or social attitude) recalls Besley and Ghatack (2005), where moral hazard can be solved more cheaply if employer and employee have the same motivation.

[15] In principle, the existence of the periodical meetings might be sufficient to sustain cooperation if the policyholders' meeting leads to fully effective peer monitoring among members of the pool. The punishment for deviation could be exclusion from the mutual agreement.

9 Appendix

9.1 Proof of Lemma 1

From the Envelope theorem $max_e U(e^{*NC}(q), q) = U^*(q)$. Therefore, at the Nash symmetric equilibrium ($e_{-i} = e_i^{BR} = e^{*NC}\ \forall i$), the FOC of (13) with respect to q can be written as:

$$\sum_{k=0}^{n-1} b(k; n-1; p) \left\{ pU' \left[W_L^{NC}\right] \frac{n-(k+1)}{n} L(e^{*NC}) \right\}$$
$$= \sum_{k=0}^{n-1} b(k; n-1; p) \left\{ (1-p)U' \left[W_0^{NC}\right] \frac{k}{n} L(e^{*NC}) \right\}$$

where $W_L^{NC} = w - \left(\frac{q(k+1)}{n} + 1 - q\right) L(e^{*NC})$ and $W_0^{NC} = w - \frac{qk}{n} L(e^{*NC})$.

The left hand side equals 0 for $k = n-1$ and the right hand side equals 0 for $k = 0$, thus:

$$p \sum_{k=0}^{n-2} b(k; n-1; p) \left\{ U' \left[W_L^{NC}\right] \frac{n-(k+1)}{n} L(e^{*NC}) \right\}$$
$$= (1-p) \sum_{k=1}^{n-1} b(k; n-1; p) \left\{ U' \left[W_0^{NC}\right] \frac{k}{n} L(e^{*NC}) \right\}$$

Expressing the binomial distribution gives:

$$p \sum_{k=0}^{n-2} \frac{(n-1)!}{k!(n-1-k)!} p^k (1-p)^{n-1-k} \left\{ U' \left[W_L^{NC}\right] \frac{n-(k+1)}{n} L(e^{*NC}) \right\}$$
$$= (1-p) \sum_{k=1}^{n-1} \frac{(n-1)!}{k!(n-1-k)!} p^k (1-p)^{n-1-k} \left\{ U' \left[W_0^{NC}\right] \frac{k}{n} L(e^{*NC}) \right\}$$
$$\Leftrightarrow \frac{L(e^{*NC})}{n} \sum_{k=0}^{n-2} \frac{(n-1)!}{k!(n-1-(k+1))!} p^{k+1} (1-p)^{n-1-k} \left\{ U' \left[W_L^{NC}\right] \right\}$$
$$= \frac{L(e^{*NC})}{n} \sum_{k=1}^{n-1} \frac{(n-1)!}{(k-1)!(n-1-k)!} p^k (1-p)^{n-k} \left\{ U' \left[W_0^{NC}\right] \right\}$$

Recalling the expression of W_0^{NC} and W_L^{NC} we therefore have:

$$\sum_{k=0}^{n-2} \frac{(n-1)!}{k!(n-1-(k+1))!} p^{k+1} (1-p)^{n-1-k} \left\{ U' \left[w - \left(\frac{q(k+1)}{n} + 1 - q \right) L(e^{*NC}) \right] \right\}$$
$$= \sum_{k=1}^{n-1} \frac{(n-1)!}{(k-1)!(n-1-k)!} p^k (1-p)^{n-k} \left\{ U' \left[w - \frac{qk}{n} L(e^{*NC}) \right] \right\}$$

That is, after a change of index with respect to the binomial probability on the left side:

$$\sum_{k=1}^{n-1}\frac{(n-1)!}{(k-1)!(n-1-k))!}p^k(1-p)^{n-k}\left\{U'\left[w-\left(\frac{qk}{n}+1-q\right)L(e^{*NC})\right]\right\}$$
$$=\sum_{k=1}^{n-1}\frac{(n-1)!}{(k-1)!(n-1-k)!}p^k(1-p)^{n-k}\left\{U'\left[w-\frac{qk}{n}L(e^{*NC})\right]\right\}$$

We thus end up with $U'\left[w-\left(\frac{qk}{n}+1-q\right)L(e^{*NC})\right]=U'\left[w-\frac{qk}{n}L(e^{*NC})\right]$ $\forall k$ which implies that $q^{*NC}=1$.

9.2 Proof of Lemma 3

In both cases, we want to prove that $EU(e^*,e^*,n)$ is increasing in n. Now, $e^{*NC}=\max_e EU(e,e^{*NC},n)$ and $e^{*C}=\max_e EU(e,e,n)$. Therefore, using the envelope theorem, we can simply analyze how the function $EU(e,e,n)$ changes with n, whatever the level of e.

Let us first define $\tilde{x}_i$ as the stochastic wealth of individual i given that k agents among the $n-1$ others suffered the loss. Therefore, $\tilde{x}_i=w-\tilde{L}_i$ where $L_i=\frac{k+1}{n}L\left(e\right)$ with probability p and $\frac{k}{n}L\left(e\right)$ with probability $(1-p)$.

$EU(e,e,n)$ is then increasing in n if $EU(e,e,n+1)$ is less risky than $EU(e,e,n)$, that is if adding a new member to the pool makes the aggregate risk decrease. This can be written as: $\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}$ less risky than $\sum_{i=1}^{n-1}\frac{\tilde{x}_i}{n-1}$. Or:

$$\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}+\tilde{\varepsilon}=\sum_{i=1}^{n-1}\frac{\tilde{x}_i}{n-1} \tag{26}$$

with $E\left[\tilde{\varepsilon}\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]=0$. Equation (26) can be rewritten as:

$$\tilde{\varepsilon}=\frac{n\sum_{i=1}^{n-1}\tilde{x}_i-(n-1)\sum_{i=1}^{n}\tilde{x}_i}{n\left(n-1\right)}=\frac{\sum_{i=1}^{n}\tilde{x}_i-n\tilde{x}_n}{n\left(n-1\right)}$$

Therefore,

$$E\left[\tilde{\varepsilon}\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]=0\text{ if and only if }\frac{\sum_{i=1}^{n}\tilde{x}_i}{n\left(n-1\right)}=\frac{1}{n-1}E\left[\tilde{x}_n\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]$$

That is if and only if:

$$E\left[\tilde{x}_n\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]=\frac{\sum_{i=1}^{n}\tilde{x}_i}{n}$$

Now, as the $\tilde{x}_i$ are i.i.d., we have:

$$E\left[\tilde{x}_n\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]=E\left[\tilde{x}_k\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]\ \forall k$$

and

$$E\left[\tilde{x}_n\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]=\frac{\sum_{k=1}^{n}E\left[\tilde{x}_k\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]}{n}=E\left[\frac{\sum_{k=1}^{n}\tilde{x}_k}{n}\left|\sum_{i=1}^{n}\frac{\tilde{x}_i}{n}\right.\right]=\frac{\sum_{k=1}^{n}\tilde{x}_k}{n}$$

9.3 Proof of Proposition 1

Let us prove that the left hand side of (25) crosses the right hand size from above when n increases.

- For $n = 1$ it is clear that inequality (25) holds (since the sole policyholder will exert the optimal effort in this case).
- However when $n \to \infty$ the impact of one policyholder's effort on the premium is negligible so that deviation is always profitable. This can be seen by rewriting inequality (25) with $n \to \infty$:

$$\frac{1}{1-\delta}\left[U(w-pL\left(e^{*C}\right))-C\left(e^{*C}\right)\right] \geq \qquad (27)$$
$$\frac{1}{1-\delta\left(1-p\right)}\left[U(w-pL\left(e^{*C}\right))+\frac{p\delta}{1-\delta}U(w-pL\left(0\right))\right]$$

Rearranging we can write:

$$C\left(e^{*C}\right) \leq p\delta\left[U(w-pL\left(0\right))-\frac{U(w-pL\left(e^{*C}\right))}{1-\delta\left(1-p\right)}\right]$$

Since $\frac{1}{1-\delta(1-p)} > 1$ and $U(w-pL\left(0\right)) < U(w-pL\left(e^{*C}\right))$, the r.h.s. of the previous inequality is negative so that the latter is never satisfied. Thus, deviation is always profitable for $n \to \infty$ and (25) does not hold.

Therefore there is a threshold value for n above which cooperation is unenforceable.

9.4 Proof of Remark 4

Let us prove Remark 4 in the case of non-cooperative effort based on equation (14) (the proof in the cooperative case, based on equation (19), is very similar).

e^{*NC} is implicitly defined by equation (14), that is in the case of CRRA preferences:

$$f(e,\gamma) \equiv \sum_{k=0}^{n-1} b(k,n-1,p)\left\{-\frac{p}{n}L'(e)U'\left(\omega-\frac{k+1}{n}L(e)\right)\right\}-C'(e)=0$$
$$\equiv \sum_{k=0}^{n-1} b(k,n-1,p)\left\{-\frac{p}{n}L'(e)\left(\omega-\frac{k+1}{n}L(e)\right)^{-\gamma}\right\}-C'(e)=0$$

Therefore, according to the theorem of implicit functions: $\dfrac{\partial e^{*NC}}{\partial \gamma} = -\dfrac{\partial f(e,\gamma)/\partial \gamma}{\partial f(e,\gamma)/\partial e}$

Now,

$$\frac{\partial f(e,\gamma)}{\partial e} = \underbrace{-C''(e)}_{<0}+\sum_{k=0}^{n-1} b(k,n-1,p)\left\{\underbrace{-\frac{p}{n}L''(e)U'(.)}_{\leq 0 \text{ if } L''(e)\geq 0}+\underbrace{\frac{p}{n}(L'^2\frac{k+1}{n}U''(.)}_{<0}\right\}$$
$$< 0 \text{ if } L''(e) \geq 0$$

And,

$$\frac{\partial f(e,\gamma)}{\partial \gamma} = \sum_{k=0}^{n-1} b(k,n-1,p)\left\{\frac{p}{n}L'(e)\ln\left(\omega-\frac{k+1}{n}L(e)\right)\left(\omega-\frac{k+1}{n}L(e)\right)^{-\gamma}\right\}$$

Therefore, $\dfrac{\partial f(e,\gamma)}{\partial \gamma} \leq 0$ if $\omega - \dfrac{k+1}{n}L(e) \geq 1 \; \forall k = 0..(n-1)$ that is if $\omega - L(e) \geq 1$, and the first part of remark 4 holds.

In the case of CARA preferences $U(W) = -\dfrac{1}{\alpha}e^{-\alpha W}$ and equation (14) can be written as

$$f(e,\alpha) \equiv \sum_{k=0}^{n-1} b(k, n-1, p) \left\{ -\frac{p}{n} L'(e) U' \left(\omega - \frac{k+1}{n} L(e) \right) \right\} - C'(e) = 0$$

with again $\dfrac{\partial f(e,\alpha)}{\partial e} < 0$ if $L''(e) \geq 0$ and

$$\frac{\partial f(e,\alpha)}{\partial \alpha} = \sum_{k=0}^{n-1} b(k, n-1, p) \left\{ \frac{p}{n} L'(e) \left(\omega - \frac{k+1}{n} L(e) \right) U' \left(\omega - \frac{k+1}{n} L(e) \right) \right\} < 0$$

which gives the second part of remark 4.

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